Radiation of interacting charges

The instantaneous power of dipole radiation produced by a system of charged particles with an instantaneous total dipole moment **d** is

$$P = \frac{2}{3c^3}\left|\frac{\mathrm{d}^2\mathbf{d}}{\mathrm{d}t^2}\right|^2,$$

were c is the speed of light. Consider a pair of particles with masses m_1 and m_2, charges q_1 and q_2, total energy E and total angular momentum $M = \mu r^2 \mathrm{d}\phi/\mathrm{d}t$ written in polar coordinate system (r, ϕ) centered at the position of the system's center of mass; $\mu = m_1 m_2/(m_1+m_2)$ is the reduced mass.

a) Calculate the dipole moment **d** and the instantaneous radiated power of the two-charge system as a function of r [2.5 points]

b) Suppose that the interaction is attractive and such that the particles undergo continuous elliptical motion. The motion can then be described as the motion of a single particle with mass μ along an elliptical trajectory given by

$$1+\varepsilon\cos\phi = \frac{a(1-\varepsilon^2)}{r},$$

where

$$a = \frac{1}{2}\left|\frac{q_1 q_2}{\mathsf{E}}\right| \quad \text{and} \quad \varepsilon = \sqrt{1-\frac{2|\mathsf{E}|M^2}{\mu(q_1 q_2)^2}}.$$

Calculate the average power radiated by the particles (in terms of E and M). [2.5 points]

c) Consider next two colliding particles that repel each other ($q_1 q_2 > 0$). The trajectory in question is now hyperbolic and given by

$$1-\varepsilon\cos\phi = \frac{a(1-\varepsilon^2)}{r}.$$

Calculate the total energy radiated by the system, assuming that at infinity, the relative velocity of the particles is v_0 and the distance between the two parallel lines along which the particles move is ρ. Evaluate this energy for $\rho = 0$. Note that $\mathsf{E} = \mu v_0^2/2$ and $M = \mu\rho v_0$. [5 points]

Mathematical help:

$$\lim_{x\to 1}\left[\frac{1}{(x^2-1)^{5/2}}\left((2+x^2)\cos^{-1}\left(\frac{1}{x}\right)-3\sqrt{x^2-1}\right)\right] = \frac{4}{15}.$$
